TURBULENT TO LAMINAR TRANSITION OF A BOUNDARY LAYER UNDER LARGE NEGATIVE PRESSURE GRADIENTS

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Results are shown of an experimental study concerning the development of a laminar sublayer in a turbulent boundary layer under large negative longitudinal pressure gradients.

Many studies were made of the reversal in a turbulent boundary layer, i.e., of the transition from a turbulent boundary layer to a laminar one under large negative pressure gradients [1-4].

The mechanism of this transition has hardly been explored, however. Here the results will be shown of a study concerning the characteristics of a laminar sublayer in a turbulent boundary layer under large negative pressure gradients, the occurrence of such a transition being hypothetically related to a situation

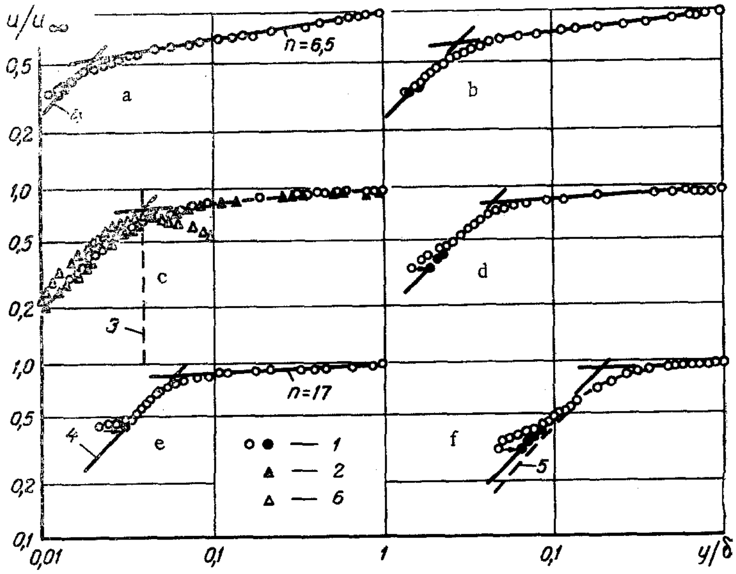

Fig. 1. Velocity profile under a negative pressure gradient: measured with a total-pressure Pitot tube (open circles represent values without a correction for the effects of viscosity, wall proximity, and velocity gradient; black dots represent values including the correction) (1), measured with a thermoanemometer (2), edge of the laminar sublayer (3), based on local values of friction (4), theoretical profile according to Blasius for $dp/dx = 0$ (5), $\sqrt{\overline{u'^2}}$ (6), $F = 0$ (a), $-1.328 \cdot 10^{-6}$ (b), $-2.21 \cdot 10^{-6}$ (c), $-2.35 \cdot 10^{-6}$ (d), $-4.37 \cdot 10^{-6}$ (e), $-10.38 \cdot 10^{-6}$ (f), $u_\infty \approx 17$ m/sec (*a*, b, c, d, e), $= 10.2$ m/sec (f).

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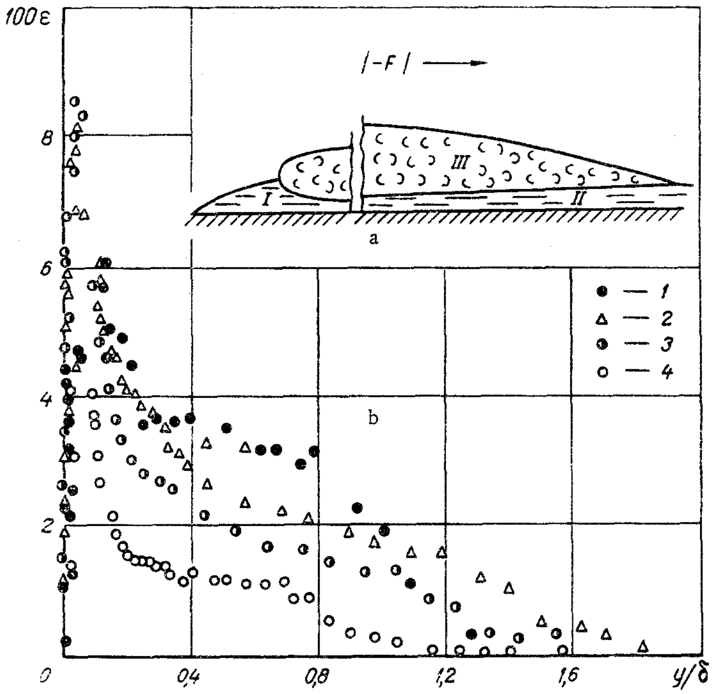

Fig. 2. (a) Fluctuations of the longitudinal velocity component across the thickness of a boundary layer under a negative pressure gradient. (b) Typical trend in the development of a boundary layer under large negative pressure gradients. $F = 0$ (1), $-1.79 \cdot 10^{-6}$ (2), $-3.02 \cdot 10^{-6}$ (3), $-6.3 \cdot 10^{-6}$ (4), laminar boundary layer (I), laminar sublayer of a turbulent boundary layer (II) turbulent core of a boundary layer (III).

where the flow in the sublayer is nearly laminar even while the gas stream is fully turbulent. Measurements in a 1 m long boundary layer at a flat plate were made with a model ÉTAM-3A thermoanemometer and with total-pressure Pitot microtubes. In the latter case, the pressure drop $P' = P_0 - P_S$ was picked off a precision alcohol manometer and read out automatically with photodiodes, optical lenses, and a relay, accurately within 0.01 mm H_2O. A longitudinal pressure gradient along the plate was produced by placing into the active zone of an aerodynamic tunnel (Fig. 4) special inserts designed so as to eliminate the effect of previous history on the development of the boundary layer [5]. The tests were performed at either almost constant or quite variable gradients dp/dx along the plate. Upstream before the insert a velocity profile of a fully turbulent boundary layer was always attained by means of a turbulizer at the front edge.

The velocity profile of the turbulent boundary layer in our test is shown in Fig. 1 as a power function of the negative longitudinal pressure gradient (absolute value):

$$\frac{u}{u_\infty} = (y/\delta)^{1/n}. \tag{1}$$

Power laws describe the measured velocity profile in the outer region of a boundary layer at all test values of the pressure gradient. At large pressure gradients, however, not only the exponent n in (1) increases but also the relative sublayer thickness increases sharply. While $\delta_1/\delta = 0.02$ at $F = 0$ in our test, for example, the relative thickness of the laminar sublayer increased by an order of magnitude to $\delta_1/\delta = 0.2$ at $F = -10.38 \cdot 10^{-6}$. The edge of the laminar sublayer, with a longitudinal pressure gradient present in the boundary layer, was related to the maximum fluctuation of the longitudinal velocity component (Fig. 1c). It has been established that, under moderate longitudinal pressure gradients, the maximum velocity fluctuations in the boundary layer correspond to the intersection points of two straight lines describing the measured velocity profile $u/u_\infty = f(y/\delta)$ (to a logarithmic scale) of the laminar sublayer and of the turbulent core respectively.

Distortions of the measured velocity profile at the wall and at the outer edge of the boundary layer

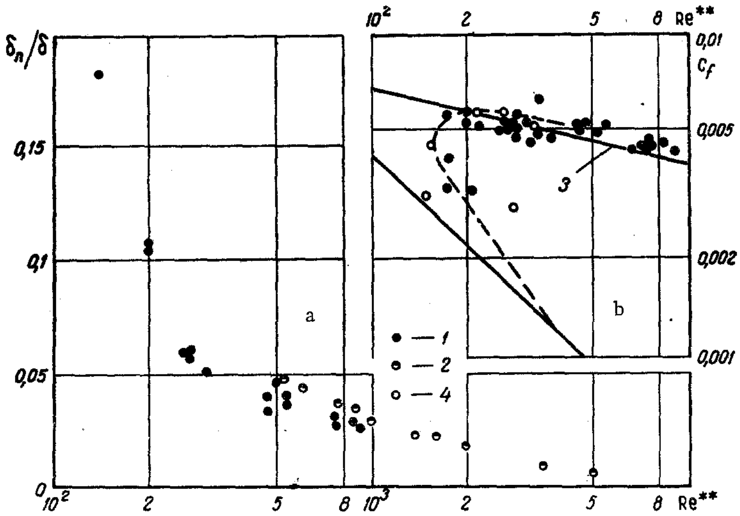

Fig. 3. Relative thickness of laminar sublayer δ_1/δ (a) and coefficient of skin-friction c_f (b) as functions of the Reynolds number Re**: $F < 0$ (1), $F = 0$ (2), calculations for $dp/dx = 0$ (3), test values for $dp/dx < 0$ (4).

under large negative pressure gradients make this profile approach the velocity profile of a laminar boundary layer, as is evident from the comparison in Fig. 1f between our measured profile and the Blasius profile. The velocity fluctuations in the boundary layer also decrease under the influence of a negative pressure gradient (Fig. 2a).

It is to be noted that the variation in δ_1/δ is uniquely determined by the value of the Re** number in a stream with either a zero or a negative pressure gradient (Fig. 3a). Inasmuch as variations in the drag depend largely on the thickness variations of the laminar sublayer, the practical significance here is that a flow without and with pressure gradients can be analyzed on a more general basis. This is brought out by a comparison between local values of the skin-friction coefficient at negative pressure gradients based on the measured velocity profile and those based on the empirical Ludwieg–Tillman formula [6]

$$c_f = 0.246 \cdot 10^{-0.678H}\,\mathrm{Re}^{**^{-0.268}},$$

for the case $dp/dx = 0$ (Fig. 3b). At rather small pressure gradients the measured values of c_f increase according to the turbulence laws for $dp/dx = 0$, while the Re** number decreases because of a negative pressure gradient in the stream. As this gradient increases, coefficient c_f begins to decrease and its values fit on the curve representing a transition from laminar to turbulent flow in the boundary layer.

As a consequence of the increasing relative thickness of the laminar sublayer under negative pressure gradients, the form factor H of the velocity profile does not decrease (as is assumed in semiempirical theories of the boundary layer) even while the profile becomes flatter in the outer region of the boundary layer, but remains almost constant instead (Fig. 4). This is possibly somewhat due to the history of the boundary layer, the exclusion of which from the analysis causes serious problems. At large negative pressure gradients ($\Delta \approx -0.02$), when reversal already begins, the value of H increases appreciably.

If a power law describing the measured velocity profile in the outer region of the boundary layer is assumed valid up to the wall, i.e., if the existence of a laminar sublayer at the wall is disregarded, then the values of H obtained at negative pressure gradients decrease and agree with those according to J. Nikuradze [7], E. Gruschwitz [8], and A. Buri [9], who did not include the laminar sublayer in their measurements.

According to the test data presented here, the transition of a turbulent boundary layer into a laminar one begins at an F number somewhere between $-3.31 \cdot 10^{-6}$ and $-3.7 \cdot 10^{-6}$ (corresponding to $\Delta = -0.0241$ and -0.293 respectively). It is to be noted that these values of F agree with those given in [3, 4], although in [4] the reversal effect was determined under different conditions: from the decrease of the heat transfer coefficient in nozzles.

The development of a turbulent to laminar transition of the boundary layer at $dp/dx \ll 0$ is shown in Fig 2b in a simplified version. At large negative pressure gradients, according to this model, the

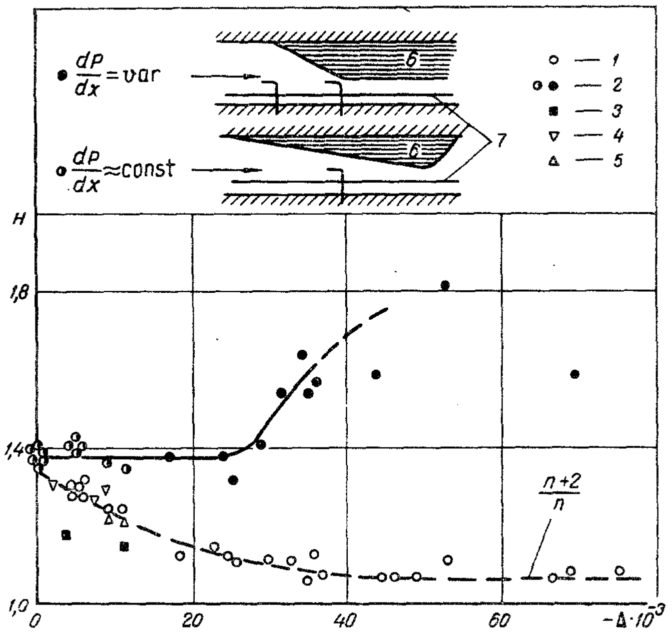

Fig. 4. Effect of negative pressure gradient on the value of the form factor H: with the laminar sublayer disregarded (1), with the laminar sublayer taken into account (2), according to tests in [7] (3), according to tests in [9] (4), according to tests in [8] (5), insert (6), plate (7).

turbulent core of a boundary layer degenerates because of the increasing relative thickness of the laminar sublayer and the simultaneously decreasing total thickness of the boundary layer.

NOTATION

c_f	is the local coefficient of skin friction;
$Re^{**} = u\delta^{**}/\nu$	is the Reynolds number referred to the momentum thickness;
$F = \nu(dp/dx)/\rho u_\infty^3$;	
$\Delta = \nu(dp/dx)/\rho u_*^3$	are the parameters of the longitudinal pressure gradient;
$H = \delta^*/\delta^{**}$	is the form factor of the velocity profile;
dp/dx	is the pressure gradient;
P_0	is the total pressure;
P_s	is the static pressure
u	is the velocity;
$u_* = \sqrt{\tau_w/\rho}$	is the dynamic velocity;
$\varepsilon = \sqrt{u'^2}/u_\infty$	is the intensity of velocity fluctuations in a boundary layer;
δ	is the thickness of boundary layer;
δ^*	is the displacement thickness of boundary layer;
δ^{**}	is the momentum thickness of boundary layer;
δ_1	is the thickness of laminar sublayer;
y	is the normal distance from the plate surface;
ν	is the kinematic viscosity of the gas;
ρ	is the density of the gas.

Subscripts

w	refers to the wall;
∞	refers to the outer edge of boundary layer.

LITERATURE CITED

1. A. A. Gukhman, A. F. Gandel'son, T. K. Katsnel'son, and B. A. Kader, in: Heat and Mass Transfer [in Russian], Izd. Énergiya, Moscow (1968), Vol. 1.

2. M. A. Badry Narayanan and V. Ramjel, J. Fluid Mech., 31, 609 (1968).
3. V. C. Patel, M. R. Head, J. Fluid Mech., 34, 2 (1968).
4. P. M. Moretti and W. M. Kays, Internatl. J. Heat and Mass Transfer, 8, 1187 (1965).
5. J. E. Nash, Aeronaut. Res. Council Current Papers, 835 (1966).
6. H. Ludweig and W. Tillman, NASA Tech. Mem., 1285 (1950).
7. J. Nikuradze, VDI Forschungsheft [German], 365 (1932).
8. E. Gruschwitz, Ing.-Archiv [German], 2, 321 (1931).
9. A. Buri, Doctoral Dissert. [Swiss], Zhurich (1931).